Example 1 – Emotional wellbeing



Screenshot of my dataset in Mplus. The order of the variables is: fltdpr wrhpp enjlf fltsd fltanx fltpcfl IL CPI cntryID



The prepared dataset is available upon request by email (S.Jak@uva.nl)

• ALL INPUT FILES FOR EXAMPLE 1

Syntax to obtain intraclass correlations of the items:

```
DATA:
  FILE IS ESSwellbeing.dat;
VARIABLE:
 NAMES ARE
                  fltdpr wrhpp enjlf
                  fltsd fltanx fltpcfl
                  IL
                  CPI
                  cntryID;
  USEVARIABLES ARE
                  wrhpp enjlf fltpcfl
                  fltdpr fltsd fltanx ;
                  fltdpr wrhpp enjlf fltsd fltanx fltpcfl (999);
 MISSING ARE
  CLUSTER IS
                  cntryID;
ANALYSIS: TYPE IS twolevel basic;
```

Syntax to fit a null-model on the between level:

```
DATA:
  FILE IS ESSwellbeing.dat;
VARIABLE:
  NAMES ARE
                  fltdpr wrhpp enjlf
                  fltsd fltanx fltpcfl
                  ΙL
                  CPI
                  cntryID;
  USEVARIABLES ARE
                  wrhpp enjlf fltpcfl
                  fltdpr fltsd fltanx ;
  MISSING ARE
                  fltdpr wrhpp enjlf fltsd fltanx fltpcfl (999);
  CLUSTER IS
                  cntryID;
ANALYSIS: TYPE IS twolevel;
MODEL:
                  %WITHIN%
                  wrhpp with enjlf-fltanx;
                  enjlf with fltpcfl-fltanx;
                  fltpcfl with fltdpr-fltanx;
                  fltdpr with fltsd fltanx;
                  fltsd with fltanx;
                  %BETWEEN%
                  wrhpp-fltanx@0;
```

Step 2 (a measurement model within and a saturated model between):

All syntax except 'MODEL' is identical to Step 1

MODEL: %WITHIN% posWB by wrhpp* enjlf fltpcfl; posWB@1; %BETWEEN% wrhpp with enjlf-fltanx; enjlf with fltpcfl-fltanx; fltpcfl with fltdpr-fltanx; fltdpr with fltsd fltanx; fltsd with fltanx;

```
Step 3a (strong factorial invariance model):
```

(all syntax except 'MODEL' is identical to Step 1)

```
MODEL:
    %WITHIN%
    posWB by wrhpp* enjlf fltpcfl (1-3);
    posWB@1;
    negWB by fltdpr* fltsd fltanx (4-6);
    negWB@1;
    %BETWEEN%
    cposWB by wrhpp* enjlf fltpcfl (1-3);
    cposWB;
    cnegWB by fltdpr* fltsd fltanx (4-6);
    cnegWB;
    wrhpp-fltanx@0;
```

Step 3b (cluster bias model):

(all syntax except 'MODEL' is identical to Step 1)

```
MODEL:
    %WITHIN%
    posWB by wrhpp* enjlf fltpcfl (1-3);
    posWB@1;
    negWB by fltdpr* fltsd fltanx (4-6);
    negWB@1;
    %BETWEEN%
    cposWB by wrhpp* enjlf fltpcfl (1-3);
    cposWB;
    cnegWB by fltdpr* fltsd fltanx (4-6);
    cnegWB;
    wrhpp-fltanx;
```

```
DATA:
 FILE IS ESSwellbeing.dat;
VARIABLE:
                   fltdpr wrhpp enjlf
 NAMES ARE
                   fltsd fltanx fltpcfl
                   IL
                   CPI
                   cntryID;
 USEVARIABLES ARE
                  wrhpp enjlf fltpcfl
                  fltdpr fltsd fltanx
                  IL CPI ;
 MISSING ARE
                  fltdpr wrhpp enjlf
                  fltsd fltanx fltpcfl
                  IL CPI (999);
  CLUSTER IS
                 cntryID;
  BETWEEN is
                  IL CPI;
ANALYSIS: TYPE IS twolevel;
MODEL:
            %WITHIN%
            posWB by wrhpp* enjlf fltpcfl (1-3);
            posWB@1;
            negWB by fltdpr* fltsd fltanx (4-6);
            negWB@1;
            %BETWEEN%
            cposWB by wrhpp* enjlf fltpcfl (1-3);
            cposWB;
            cnegWB by fltdpr* fltsd fltanx (4-6);
            cnegWB;
            cnegWB with IL CPI;
            cposWB with IL CPI;
            IL with CPI;
            enjlf on CPI; ! direct effect of CPI on Enjoy Life
OUTPUT:
            stdyx;
```

Step 4 (adding observed between-level variables to the model):

• ANNOTATED OUTPUT FOR EXAMPLE 1

Fit statistics of Step 1 (Chi-square statistic of the null-model)



Intraclass correlations of Step 1 (proportions of indicator variance at the country-level)

Estimated Intraclass Correlations for the Y Variables

Variable	Intraclass Correlation	Variable	Intraclass Correlation	Variable	Intraclass Correlation
WRHPP FLTDPR	0.041 0.067	ENJLF FLTSD	0.068	FLTPCFL FLTANX	0.035

ICCs range from .035 to .122

Fit statistics of Step 2 (measurement model at within-level)

```
THE MODEL ESTIMATION TERMINATED NORMALLY
MODEL FIT INFORMATION
Number of Free Parameters
                                               40
Loglikelihood
         H0 Value
                                      -321348.184
         HO Scaling Correction Factor 8.4951
           for MLR
         H1 Value
                                      -320072.111
         H1 Scaling Correction Factor
                                          7.8032
           for MLR
Information Criteria
         Akaike (AIC)
                                      642776.368
         Bayesian (BIC)
                                      643132.676
         Sample-Size Adjusted BIC
                                      643005.555
           (n* = (n + 2) / 24)
Chi-Square Test of Model Fit
         Value
                                         587.562*
         Degrees of Freedom
                                              8
                                          0.0000
         P-Value
         Scaling Correction Factor
                                           4.3436
           for MLR
4
   The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used
   for chi-square difference testing in the regular way. MLM, MLR and WLSM
   chi-square difference testing is described on the Mplus website. MLMV, WLSMV,
   and ULSMV difference testing is done using the DIFFTEST option.
RMSEA (Root Mean Square Error Of Approximation)
         Estimate
                                            0.036
CFI/TLI
        CFI
                                            0.973
          TLI
                                            0.898
```

χ²₍₈₎ = 587.562, *p* < .05, RMSEA = .036, CFI = .97

Standardized parameter estimates of Step 2 (measurement model at within-level)

STANDARDIZED MODEL RESULTS

STDYX Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value		
Within Level						
POSWB BY						
WRHPP	0.777	0.010	78.896	0.000		
ENJLF	0.756	0.010	73.629	0.000		
FLTPCFL	0.621	0.011	55.228	0.000		
NEGWB BY					\mathbf{F}	Factor loadings (λ_w)
FLTDPR	0.740	0.008	88.112	0.000		
FLTSD	0.754	0.008	94.088	0.000		
FLTANX	0.627	0.015	40.834	0.000		
NEGWB WITH POSWB	-0.694	0.012	-58.237	0.000	}	Factor covariance (ϕ_w)
Variances						
POSWB	1.000	0.000	999.000	999.000	1	Easter veriences (a)
NEGWB	1.000	0.000	999.000	999.000	1	Factor variances (ϕ_w)
Residual Variances	3					
WRHPP	0.397	0.015	25.940	0.000	ר	
ENJLF	0.428	0.016	27.584	0.000		
FLTPCFL	0.615	0.014	44.079	0.000		Residual variances (A)
FLTDPR	0.453	0.012	36.420	0.000		(0_W)
FLTSD	0.432	0.012	35.808	0.000		
FLTANX	0.607	0.019	31.531	0.000		
					1	

Fit statistics of Step 3a (strong factorial invariance)

THE MODEL ESTIMATION TERMINATED NORMALLY MODEL FIT INFORMATION Number of Free Parameters 22 Loglikelihood H0 Value -324170.192 H0 Scaling Correction Factor 25.1474 for MLR H1 Value -320072.111 Information Criteria 648384.383 Akaike (AIC) Bayesian (BIC) 648580.352 Sample-Size Adjusted BIC 648510.436 (n* = (n + 2) / 24)Chi-Square Test of Model Fit Value 8196.160* Degrees of Freedom 26 0.0000 P-Value Scaling Correction Factor Undefined for MLR The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option. RMSEA (Root Mean Square Error Of Approximation) 0.076 Estimate CFI/TLI CFI 0.618 TLI 0.559

 $\chi^{2}_{(26)}$ = 8196.16, *p* < .05, RMSEA = .076, CFI = .62

Fit statistics of Step 3b (accounting for country-bias by freeing residual variance at the country-level)

THE MODEL	ESTIMATION TERMINATED NORMAL	LY	
MODEL FIT	INFORMATION		
Number of	Free Parameters	28	
Loglikeli	hood		
	H0 Value H0 Scaling Correction Factor for MLR	-321361.991 11.7680	
	H1 Value	-320072.111	
	H1 Scaling Correction Factor for MLR	7.8032	
Informati	on Criteria		
	Akaike (AIC)	642779.982	
	Bavesian (BIC)	643029.397	
	Sample-Size Adjusted BIC (n* = (n + 2) / 24)	642940.413	
Chi-Squar	e Test of Model Fit		
	Value Degrees of Freedom P-Value Scaling Correction Factor for MLR	1145.311* 20 0.0000 2.2525	
* The c for c chi-s and U	hi-square value for MLM, MLMV hi-square difference testing quare difference testing is d LSMV difference testing is do	y, MLR, ULSMV, in the regula: lescribed on the one using the h	WLSM and WLSMV cannot be used r way. MLM, MLR and WLSM he Mplus website. MLMV, WLSMV, DIFFTEST option.
RMSEA (Ro	ot Mean Square Error Of Appro	eximation)	
	Estimate	0.032	
CFI/TLI			
	CFI TLI	0.947 0.921	

 $\chi^{2}_{(20)}$ = 1145.3, *p* < .05, RMSEA = .031, CFI = .95

Parameter estimates of Step 3b (unstandardized)

MODEL RESULTS					
	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	
Within Level					
POSWB BY					7
WRHPP	0.626	0.013	47.612	0.000	
ENJLF	0.632	0.011	55.332	0.000	
FLTPCFL	0.507	0.012	43.851	0.000	
					- Factor loadings (λ_{-})
NEGWB BY					i detor roddings (n _w)
FLTDPR	0.510	0.013	39.830	0.000	
FLIDD	0.517	0.015	25.060	0.000	
LINN	0.111	0.017	23.909	0.000	
NEGWB WITH					
POSWB	-0.694	0.012	-58.233	0.000	F actor covariance (α)
					j Tactor covariance (ψ_w)
Variances		_			
POSWB	1.000	0.000	999.000	999.000	
NEGWB	1.000	0.000	999.000	999.000	Factor variances (ϕ_w)
Residual Varianc	es				
WRHPP	0.258	0.010	26.483	0.000	1
ENJLF	0.299	0.015	19.981	0.000	
FLTPCFL	0.411	0.014	28.778	0.000	Residual variances (A)
FLTDPR	0.216	0.011	20.150	0.000	$\mathbf{Residual Valiances}\left(0_{W}\right)$
FLTSD	0.203	0.007	28.228	0.000	
FLTANX	0.304	0.016	19.300	0.000	
Between Level					
CPOSWB BY					
WRHPP	0.626	0.013	47.612	0.000	ר ר
ENJLF	0.632	0.011	55.332	0.000	
FLTPCFL	0.507	0.012	43.851	0.000	
					Easter loadings ())
CNEGWB BY					$-$ Factor loadings (Λ_b)
FLTDPR	0.510	0.013	39.830	0.000	
FLTSD	0.517	0.015	34.586	0.000	
FLIANX	0.444	0.017	25.969	0.000	
CNEGWB WITH					
CPOSWB	-0.080	0.020	-4.098	0.000	b Eactor covariance (α)
					F Pactor covariance (ψ_b)
Intercepts					-
WRHPP	2.889	0.031	93.143	0.000	1
ENJLF	2.860	0.042	68.071	0.000	
FLTPCFL	2.794	0.029	96.094	0.000	- Intercepts (α_{1})
FLIDPK	1.526	0.034	44.322	0.000	
FLTANX	1.666	0.049	33.987	0.000	
	1.000	0.015	001007	0.000	
Variances					
CPOSWB	0.062	0.015	4.015	0.000	Factor variances (a)
CNEGWB	0.137	0.034	4.082	0.000	• Factor variances (ψ_b)
Residual Variance	29				
WRHPP	0,006	0.002	2.460	0.014	1
ENJLF	0.014	0.005	2.991	0.003	
FLTPCFL	0.014	0.004	3.887	0.000	
FLTDPR	0.004	0.003	1.337	0.181	• Residual variances (θ_{L})
FLTSD	0.004	0.004	1.099	0.272	(-0)
FLTANX	0.039	0.010	3.854	0.000	

Calculating the ICC of the common factors using the factor variances: $ICC = \phi_b / (\phi_b + \phi_w)$ Positive wellbeing : .062/ (.062 + 1) = .058 Negative wellbeing : .137/ (.137 + 1) = .120

Calculating the proportion of country-bias per item:

Proportion of country-level bias in country-level variance:

 $\theta_{\rm b}$ / ($\lambda_{\rm b}^2 \phi_{\rm b} + \theta_{\rm b}$)

For Item 2: $.014 / (.632^2 * .062 + .014) = .361$ For Item 3: $.014 / (.507^2 * .062 + .014) = .468$ For Item 6: $.039 / (.444^2 * .137 + .039) = .591$

Proportion of country-level bias in total variance:

 θ_{b} / ($\lambda_{b}^{2} \varphi_{b} + \theta_{b} + \lambda_{w}^{2} \varphi_{w} + \theta_{w}$).

For Item 2: $.014 / (.632^2 * .062 + .014 + .632^2 + .299) = .019$ For Item 3: $.014 / (.507^2 * .062 + .014 + .507^2 + .411) = .020$ For Item 6: $.039 / (.444^2 * .137 + .039 + .444^2 + .304) = .068$

Parameter estimates of Step 3b. (standardized)

STDYX Standardiza	tion				
				Two-Tailed	
	Estimate	S.E.	Est./S.E.	P-Value	
Within Level					
POSWB BY					ר
WRHPP	0.777	0.010	78.674	0.000	
ENJLF	0.756	0.010	73.763	0.000	
FLTPCFL	0.621	0.011	55.249	0.000	
NECUE BY					Factor loadings (λ_w)
FLTDPR	0.740	0.008	88,110	0.000	e (w/
FLTSD	0.754	0.008	94.320	0.000	
FLTANX	0.627	0.015	40.842	0.000	
NEGWB WITH					• - • • • • •
POSWB	-0.694	0.012	-58.233	0.000	Factor correlation (ϕ_{w})
					• (TW)
POSWB	1 000	0 000	000 000	000 000	
NEGWB	1.000	0.000	999 000	999 000	Factor variances (a)
MEGNE	1.000	0.000	555.000	555.000	\downarrow Factor variances (ψ_{w})
Residual Varianc	es				
WRHPP	0.397	0.015	25.877	0.000	1
ENJLF	0.428	0.016	27.609	0.000	
FLTPCFL	0.615	0.014	44.122	0.000	Residual variances (θ_{m})
FLTDPR	0.453	0.012	36.479	0.000	
FLTSD	0.432	0.012	35.841	0.000	
FLIANX	0.607	0.019	31.532	0.000	
					-
Between Level					
Between Bever					
CPOSWB BY					-
WRHPP	0.894	0.037	24.405	0.000	
ENJLF	0.801	0.072	11.086	0.000	
FLTPCFL	0.727	0.066	11.037	0.000	
CNECKB BY					Factor loadings ($\lambda_{\rm b}$)
FLTDPR	0.949	0.033	29.050	0.000	
FLTSD	0.949	0.047	20.287	0.000	
FLTANX	0.640	0.081	7.873	0.000	
CNEGWB WITH					
CPOSWB	-0.871	0.067	-12.957	0.000	Factor correlation (ϕ_b)
Intercente					
WRHPP	16.564	2.014	8,223	0.000	ר – ר
ENJLF	14.570	1.108	13.153	0.000	
FLTPCFL	16.079	1.327	12.117	0.000	L Intercents (α)
FLTDPR	7.670	0.915	8.382	0.000	Γ intercepts (u_b)
FLTSD	8.123	0.820	9.907	0.000	
FLTANX	6.487	0.450	14.430	0.000	
Variances					
CPOSWB	1.000	0.000	999.000	999.000	
CNEGWB	1.000	0.000	999.000	999.000	Factor variances (ϕ_b)
Residual Variand	ces	<u> </u>			1
WRHPP	0.201	0.065	3.069	0.002	
FITECET	0.358	0.116	3.090	0.002	
FLTDPR	0.100	0,062	1.520	0.105	• Residual variances (θ_i)
FLTSD	0.100	0.089	1.122	0.262	
FLTANX	0.591	0.104	5.688	0.000	

Fit statistics of Step 4 (Adding observed variables)

THE MODEL ESTIMATION TERMINATED NORMALLY WARNING: THE RESIDUAL COVARIANCE MATRIX (THETA) IS NOT POSITIVE DEFINITE. THIS COULD INDICATE A NEGATIVE VARIANCE/RESIDUAL VARIANCE FOR AN OBSERVED VARIABLE, A CORRELATION GREATER OR EQUAL TO ONE BETWEEN TWO OBSERVED VARIABLES, OR A LINEAR DEPENDENCY AMONG MORE THAN TWO OBSERVED VARIABLES. CHECK THE RESULTS SECTION FOR MORE INFORMATION. MODEL FIT INFORMATION Warning appears because the Number of Free Parameters 37 Loglikelihood As it is not significant from H0 Value -321571.591 H0 Scaling Correction Factor 9.1453 for MLR as absence of cluster bias in H1 Value -320275.414 H1 Scaling Correction Factor 6.0132 this item. for MLR Information Criteria Akaike (AIC) 643217.181 Bayesian (BIC) 643546.819 Sample-Size Adjusted BIC 643429.232 (n* = (n + 2) / 24)Chi-Square Test of Model Fit 1383.132* Value Degrees of Freedom 28 **P-Value** 0.0000 Scaling Correction Factor 1.8743 for MLR The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option. RMSEA (Root Mean Square Error Of Approximation) Estimate 0.030 CFI/TLI CFI 0.952 TLI 0.926

χ²₍₂₈₎ = 1383.1, *p* < .05, RMSEA = .030, CFI = .95

country-level residual variance of the item FLTSD is negative. zero this can just be interpreted

					Two-Tailed		
		Estimate	S.E.	Est./S.E.	P-Value		
ſ							
	Within Level						
4							
	POSWB BY					_	
	WRHPP	0.626	0.013	47.625	0.000		
	ENJLF	0.632	0.011	55.305	0.000		
	FLTPCFL	0.507	0.012	43.860	0.000		
	NEGWB BY						Eactor loadings (λ_{-})
	FLTDPR	0.510	0.013	39.771	0.000		(\mathcal{M}_{W})
	FLTSD	0 518	0.015	34 638	0.000		
	ET TANY	0.010	0.013	25 000	0.000		
	FLIANA	0.111	0.017	25.900	0.000	J	
	NEGWB WITH					1	$\mathbf{E}_{\mathbf{r}}$
	POSWB	-0.694	0.012	-58.211	0.000	_ 5 _	Factor covariance (ϕ_w)
	Variances						
	POSWB	1.000	0.000	999.000	999.000	1	$\mathbf{E}_{\mathbf{r}}$
	NEGWB	1.000	0.000	999.000	999.000		Factor variances (ϕ_w)
	Residual Variances					_	
	WRHPP	0 258	0 010	26 470	0.000	<u>ר</u>	
	FNTE	0.200	0.010	10 005	0.000		
	FITDOFT	0.299	0.013	12.203	0.000		Residual variances (A)
	5 PI BCE P	0.411	0.014	28.775	0.000	- F	(0_w)
	FLIDPR	0.216	0.011	20.138	0.000		
	FLTSD	0.203	0.007	28.289	0.000		
	FLTANX	0.304	0.016	19.302	0.000		
ſ							
	Between Level						
L							
	CPOSWB BY						
	WRHPP	0.626	0.013	47.625	0.000		
	ENJLF	0.632	0.011	55.305	0.000		
	FLTPCFL	0.507	0.012	43,860	0.000		
	THIGTH	0.007	0.012	10.000	0.000		
	CNECHE BY						Eactor loadings (λ_{i})
	ELTODD	0 510	0 012	20 771	0.000		$1 \operatorname{actor} \operatorname{rotadings}(n_b)$
	FLIDFR	0.510	0.015	39.771	0.000		
	FLISD	0.518	0.015	34.638	0.000		
	FLTANX	0.444	0.017	25.988	0.000		
	CNEGWB WITH					-	
	INDLIB	-2.806	0.729	-3.848	0.000		
	CPI	-7.272	1.577	-4.611	0.000		
	CPOSWB	-0.085	0.020	-4.364	0.000		
							C
	CPOSWB WITH						Covariances between common
	INDLIB	1.801	0.574	3.141	0.002		factors and contextual verichles
	CPI	3.572	0.827	4.320	0.000		factors and contextual variables
	TNDLTB WITTH						
	CDT WIIN	110 010	20 264	2 002	0.005		
	UFI	110.013	33.269	2.002	0.005		
	means						
	INDLIB	74.544	2.857	26.094	0.000	L	Means of contextual variables
	CPI	62.414	3.553	17.566	0.000		wicans of contextual variables
	Intercepts						
	WRHPP	2.889	0.031	93.124	0.000	ר	
	ENJLF	2.859	0.042	68.048	0.000		
	FLTPCFL	2.794	0.029	96.027	0.000		
	FLTDPR	1.526	0.034	44.339	0.000		intercepts ($\alpha_{\rm b}$)
	FLTSD	1.638	0.040	41.059	0.000		
	FLTANX	1.666	0.049	33,988	0.000		
			2.212		5.000		
	Variances						
	TNDTTP	220.202	79 000	2 000	0.000		V
	INDLIB	220.361	/3.938	2.980	0.003		variances of common factors
	CPI	365.968	71.229	5.138	0.000		and contextual mental 1
	CPOSWB	0.061	0.015	4.087	0.000	1	and contextual variables
	CNEGWB	0.173	0.044	3.909	0.000		
	Residual Variances						
	WRHPP	0.006	0.003	2.282	0.023		
	ENJLF	0.015	0.005	3.221	0.001		
	FLTPCFL	0.013	0.004	3.612	0.000		
	FLTDPR	0.008	0.005	1.611	0.107		Residual variances (θ_{i})
							· · · · · · · · · · · · · · · · ·

Parameter estimates of Step 4 (unstandardized)

FLTSD

FLTANX

0.000

0.034

0.003 -0.090 0.928

3.844

0.000

0.009

Two-Tailed S.E. Est./S.E. Estimate **P-Value** Within Level POSWB BY WRHPP 0.777 0.010 78.681 0.000 0.010 73.769 0.756 0.000 ENJLF FLTPCFL 0.621 0.011 55.243 0.000 Factor loadings (λ_w) NEGWB BY FLTDPR 0.739 0.008 87.692 0.000 0.754 0.008 94.948 0.000 FLTSD FLTANX 0.627 0.015 40.882 0.000 NEGWB WITH POSWB -0.694 0.012 -58.211 0.000 } Factor correlation (ϕ_w) Variances 1.000 0.000 999.000 999.000 POSWB Factor variances (ϕ_w) NEGWB 1.000 999.000 0.000 999.000 Residual Variances WRHPP 0.397 0.015 25.879 0.000 ENJLF 0.428 0.016 27.614 0.000 FLTPCFL. 0.615 0.014 44.115 0.000 Residual variances (θ_{w}) FLTDPR 0.453 0.012 36.346 0.000 0.432 0.012 36.037 FLTSD 0.000 FLTANX 0.607 0.019 31.564 0.000 Between Level CPOSWB BY WRHPP 0.893 0.040 22.501 0.000 0.790 0.070 11.209 ENJLF 0.000 FLTPCFL 0.734 0.068 10.750 0.000 Factor loadings ($\lambda_{\rm b}$) CNEGWB BY FLTDPR 25.353 0.918 0.036 0.000 FLTSD 1,002 0.027 36.785 0.000 FLTANX 0.706 0.080 8.831 0.000 CNEGWB WITH INDLIB -0.4550.110 -4.1390.000 CPI -0.914 0.033 -27.517 0.000 CPOSWB -0.830 0.097 -8.598 0.000 Correlations between common CPOSWB WITH INDLIB 0.490 0.107 4.602 0.000 factors and contextual variables 0.755 8.072 0.000 CPI 0.094 **INDLTB** WITH 0.387 0.116 3.329 0.001 CPI Means INDLIB 5.022 0.983 5.107 0.000 Means of contextual variables CPI 3.263 0.415 7.865 0.000 Intercepts WRHPP 16.651 2.020 8.241 0.000 ENJLF 14.451 1.084 13.325 0.000 FLTPCFL 16.338 1.360 12,014 0.000 Intercepts ($\alpha_{\rm b}$) FLTDPR 6.605 0.910 7.254 0.000 FLTSD 7.630 0.851 8.964 0.000 FLTANX 6.372 0.418 15.228 0.000 Variances Variances of common factors INDLIB 1.000 0.000 999.000 999.000 CPI 1.000 0.000 999.000 999.000 and contextual variables CPOSWB 1.000 0.000 999,000 999.000 CNEGWB 1.000 0.000 999.000 999.000 Residual Variances WRHPP 0.202 0.071 2.854 0.004 ENJLF 0.376 0.111 3.379 0.001 FLTPCFL 0.461 0.100 4.601 0.000 Residual variances ($\theta_{\rm b}$) FLTDPR 0.157 0.066 2.367 0.018 FLTSD -0.005 999.000 999.000 999.000 FLTANX 0.501 0.113 4.438 0.000

Parameter estimates of Step 4 (standardized)

Country-level parameter estimates of Step 4 with the item 'Enjoy Life' regressed on CPI (unstandardized)

Between Level						
CPOSWB BY						
WRHPP	0.626	0.013	47.680	0.000		
ENJLF	0.632	0.011	55.229	0.000		
FLTPCFL	0.507	0.012	43.824	0.000		
CNEGWB BY						
FLTDPR	0.510	0.013	39.770	0.000		
FLTSD	0.518	0.015	34.639	0.000		
FLTANX	0.444	0.017	25.988	0.000		
ENJLF ON					1	Direct offect of CDI
CPI	0.004	0.001	3.370	0.001	ł	on ENJLF (b)
CNEGWB WITH						
INDLIB	-2.802	0.728	-3.850	0.000		
CPI	-7.267	1.579	-4.601	0.000		
CPOSWB	-0.075	0.017	-4.307	0.000		
CPOSWB WITH						
INDLIB	1.613	0.514	3.137	0.002		
CPI	3.064	0.726	4.219	0.000		
INDLIB WITH						
CPI	109.877	39.189	2.804	0.005		
Means						
INDLIB	74.548	2.856	26.098	0.000		
CPI	62.414	3.553	17.566	0.000		
Intercepts						
WRHPP	2,889	0.031	93.153	0.000		
ENJLF	2.634	0.075	35.352	0.000		
FLTPCFL	2.794	0.029	96.023	0.000		
FLTDPR	1.526	0.034	44.339	0.000		
FLTSD	1.638	0.040	41.059	0.000		
FLTANX	1.666	0.049	33.989	0.000		
Variances						
INDLIB	220.374	73.951	2.980	0.003		
CPI	365.965	71.230	5.138	0.000		
CPOSWB	0.053	0.013	4.152	0.000		
CNEGWB	0.173	0.045	3.867	0.000		
Residual Variances						
WRHPP	0.005	0.002	2.344	0.019		
ENJLF	0.011	0.003	3.301	0.001		
FLTPCFL	0.013	0.004	3.631	0.000		
FLTDPR	0.008	0.005	1.577	0.115		
FLTSD	0.000	0.003	-0.079	0.937		
FLTANX	0.034	0.009	3.773	0.000		

Country-level parameter estimates of Step 4 with the item 'Enjoy Life' regressed on CPI (standardized)

Direct effect of CPI

on ENJLF (β)

Between Level					
CPOSWB BY					
WRHPP	0.892	0.041	22.019	0.000	
FNITE	0.648	0.073	8 824	0,000	
FLTPCFL	0.711	0.071	9 961	0.000	
THICTH	0.711	0.071	5.501	0.000	
CNEGWB BY					
FLTDPR	0.918	0.037	24.971	0.000	
FLTSD	1.002	0.028	35.701	0.000	
FLTANX	0.706	0.082	8.658	0.000	
ENTLE ON					
CDT ON	0 200	0 077	2 050	0 000	
CPI	0.306	0.077	5.959	0.000	
CNEGWB WITH					
INDLIB	-0.454	0.110	-4.134	0.000	
CPI	-0.914	0.033	-27.393	0.000	
CPOSWB	-0.783	0.104	-7.521	0.000	
CPOSWB WITH					
INDLIB	0.470	0.104	4.525	0.000	
CPI	0.693	0.100	6.915	0.000	
CDI WITH	0 207	0 116	2 224	0 001	
CPI	0.367	0.116	3.324	0.001	
Means					
INDLIB	5.022	0.983	5.107	0.000	
CPI	3.263	0.415	7.865	0.000	
Intercepts					
WRHPP	17.797	2.074	8.580	0.000	
ENJLF	11.695	1.223	9.566	0.000	
FLTPCFL	16.953	1.335	12.699	0.000	
FLTDPR	6.611	0.928	7.125	0.000	
FLTSD	7.634	0.857	8.908	0.000	
FLTANX	6.373	0.419	15.208	0.000	
Variances	1 000	0.000	000 000	000 000	
INDLIB	1.000	0.000	999.000	999.000	
CPI	1.000	0.000	999.000	999.000	
CPOSWB	1.000	0.000	999.000	999.000	
CNEGWB	1.000	0.000	999.000	999.000	
Residual Variances					
WRHPP	0.205	0.072	2.834	0.005	
ENJLF	0.211	0.066	3.225	0.001	
FLTPCFL	0.494	0.102	4.859	0.000	
FLTDPR	0.157	0.068	2.326	0.020	
FLTSD	-0.004	999.000	999.000	999.000	
FLTANX	0.502	0.115	4.362	0.000	

Example 2 - Mathematical ability

Preparing the data for analysis in Mplus: Download the dataset from <u>https://www.oecd.org/pisa/pisaproducts/</u> Select the students who received Booklet 1 (BOOKID = 1) Select relevant variables (in this example: PM00FQ01 PM00GQ01 PM903Q03 PM909Q01 PM923Q04 PM924Q02 PM955Q01 PM995Q01 PM998Q02) Select the relevant country-level variables from the country-data (in this example: 'SC14Q02_mean' and 'SMRATIO_mean') Match the individual data to the country-level data (using the variable 'CNT') Recode all responses to numerical values Give numerical values to missing values (e.g. 999) Save dataset as .dat file without columnnames

Print screen of the prepared dataset:

Mplus - [pisa]												
	F	File	E	dit		Vie	w	М	plu	is Plot	Diagram	Window Help
D		Ê		3	X	E	۵	e		😂 RUN	$ \mathbb{M} \mathbb{M}$	1 <u>123</u> 1/2
	1	0	0	1	0	1	1	1	1	1.958	88.854	1
1	0	0	1	1	1	1	1	1	0	1.958	88.854	1
	0	0	0	1	0	1	1	1	1	1.958	88.854	1
	0	0	0	1	0	1	0	0	1	1.958	88.854	1
1	0	0	0	1	0	0	0	0	0	1.958	88.854	1
	0	0	0	1	0	0	1	0	0	1.958	88.854	1
	1	0	1	1	0	0	1	1	1	1.958	88.854	1
	0	0	0	1	0	1	1	1	1	1.958	88.854	1
	0	0	0	•	0	0	0	0	•	4 050	00 054	*

The prepared dataset is available upon request by email (S.Jak@uva.nl)

• ALL INPUT FILES FOR EXAMPLE 2

Mplus-scripts for Example 2

The item labels for v1 to v9 as used by PISA are respectively: PM00FQ01, PM00GQ01,

PM903Q03, PM909Q01, PM923Q04, PM924Q02, PM955Q01, PM995Q01 and PM998Q02.

Step 1 (obtaining intraclass correlations and fitting a null-model on the between-level):

```
DATA:
 FILE IS pisa.dat;
VARIABLE:
 NAMES ARE
                v1-v9
                 short
                  stratio
                  cntryID;
 USEVARIABLES ARE v1-v9;
 CATEGORICAL ARE v1-v9;
 MISSING ARE
                 v1-v9 (7 8 9);
  CLUSTER IS
                 cntryID;
ANALYSIS: TYPE IS twolevel basic;
         ESTIMATOR IS WLSMV;
```

```
DATA:
 FILE IS pisa.dat;
VARIABLE:
 NAMES ARE
                  v1-v9
                   short
                   stratio
                   cntryID;
  USEVARIABLES ARE
                   v1-v9;
  CATEGORICAL ARE
                   v1-v9;
 MISSING ARE v1-v9 (7 8 9);
CLUSTER IS cntryID;
ANALYSIS: TYPE IS twolevel;
          ESTIMATOR IS WLSMV;
MODEL:
            %WITHIN%
             v1 with v2-v9;
             v2 with v3-v9;
             v3 with v4-v9;
             v4 with v5-v9;
             v5 with v6-v9;
             v6 with v7-v9;
             v7 with v8-v9;
             v8 with v9;
             %BETWEEN%
             v1-v9@0;
```

Step 2 (a measurement model within and a saturated model between):

All syntax except 'MODEL' is identical to Step 1

```
MODEL: %WITHIN%
math by v1* v2-v9;
math@1;
%BETWEEN%
v1 with v2-v9;
v2 with v3-v9;
v3 with v4-v9;
v4 with v5-v9;
v4 with v5-v9;
v5 with v6-v9;
v6 with v7-v9;
v7 with v8-v9;
v8 with v9;
v1-v9;
```

Step 3a (strong factorial invariance model):

All syntax except 'MODEL' is identical to Step 1

```
MODEL: %WITHIN%
math by v1* v2-v9(1-9);
math@1;
%BETWEEN%
Bmath by v1* v2-v9 (1-9);
Bmath;
v1-v9@0;
```

DATA: FILE IS pisa.dat; VARIABLE: NAMES ARE v1-v9 short stratio cntryID; USEVARIABLES ARE v1-v9 short stratio; CATEGORICAL ARE v1-v9; BETWEEN ARE short stratio; v1-v9 (7 8 9); MISSING ARE CLUSTER IS cntryID; DEFINE: stratio = stratio/100; ANALYSIS: TYPE IS twolevel; ESTIMATOR IS WLSMV; MODEL: %WITHIN% math by v1* v2-v9 (1-9); math@1; %BETWEEN% Bmath by v1* v2-v9 (1-9); Bmath; Bmath with short stratio; short with stratio; v1-v9@0; OUTPUT: stdyx;

Step 4 (adding observed between-level variables to the model

• ANNOTATED OUTPUT FOR EXAMPLE 2

Fit statistics of Step 1 (Chi-square statistic of the null-model)

MODEL FIT INFORMATION	
Number of Free Parameters	45
Chi-Square Test of Model Fit	
Value	200.784*
Degrees of Freedom	45
P-Value	0.0000

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.

Significant chi-square indicates significant country-level variance

Estimated Intraclass Correlations for the Y Variables

Variable	Intraclass Correlation	Variable	Intraclass Correlation	Variable	Intraclass Correlation
V1	0.069	V2	0.097	V3	0.138
V4	0.075	V5	0.104	V6	0.082
V7	0.057	V8	0.101	V9	0.107

ICCs range from .057 to .138

Fit statistics of Step 2 (measurement model at within-level)

THE MODEL ESTIMATION TERMINATED NORMALLY MODEL FIT INFORMATION Number of Free Parameters 63 Chi-Square Test of Model Fit Value 36.736* Degrees of Freedom 27 P-Value 0.1001 The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option. RMSEA (Root Mean Square Error Of Approximation) 0.005 Estimate CFI/TLI CFI 0.998 ТЬI 0.995

χ²₍₂₇₎ = 36.74, *p* = .10, RMSEA = .005, CFI = .998

Standardized parameter estimates of Step 2 (measurement model at within-level)

STDYX Standardizat	ion				
	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	
Within Level					
V1 V2 V3 V4 V5 V6 V7 V8 V9	0.682 0.600 0.724 0.487 0.710 0.701 0.374 0.745 0.562	0.010 0.013 0.019 0.022 0.010 0.022 0.012 0.012	67.375 46.561 72.283 25.211 32.800 68.867 17.246 61.353 35.622	0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	Factor loadings (λ_w)
Variances MATH	1.000	0.000	999.000	999.000 }	Factor variance (ϕ_w)

Residual variances (θ_w) are fixed at 1 for scaling and not provided in the output

Fit statistics of Step 3a (strong factorial invariance)

	THE MODEL ESTIMATION T	ERMINATED NORMALLY			
	MODEL FIT INFORMATION				
	Number of Free Paramet	ers	19		
ſ	Chi-Square Test of Mod	el Fit			
	Value		84.642*		
	Degrees of F	reedom	71		
	r-vaiue		0.1204		
	 The chi-square val for chi-square dif chi-square differe and ULSMV differen 	ue for MLM, MLMV, ference testing in nce testing is des ce testing is done	MLR, ULSMV, WLSM the regular way. cribed on the Mpl using the DIFFTE	and WLSMV can MLM, MLR and us website. N ST option.	not be used 1 WLSM 4LMV, WLSMV,
ſ	RMSEA (Root Mean Squar	e Error Of Approxi	mation)		
	Estimate		0.003		
	CFI/TLI				
	CFI		0.997		
	TLI		0.997		

 $\chi^{2}_{(71)}$ = 84.64, *p* = .12, RMSEA = .003, CFI = .997

Parameter estimates of Step 3a (unstandardized)

				Two-Tailed	
	Estimate	S.E. E	Cst./S.E.	P-Value	
Within Level					
MATH BY				_	
Vl	0.921	0.025	37.594	0.000	
V2	0.756	0.026	29.603	0.000	
V3	1.053	0.033	31.474	0.000	
V4	0.561	0.027	21.170	0.000	
V5	1.012	0.062	16.268	0.000	Factor loadings ($\lambda_{\rm w}$)
V6	0.986	0.026	37.252	0.000	
V 7	0.413	0.027	15.450	0.000	
V8	1.113	0.041	26.911	0.000	
V9	0.687	0.029	23.921	0.000 -	
Variances MATH Between Level	1.000	0.000	999.000	999.000 }	Factor variance (ϕ_w)
BMATH BY					
V1	0.921	0.025	37.594	0.000	
V2	0.756	0.026	29.603	0.000	
₩3	1.053	0.033	31.474	0.000	
V4	0.561	0.027	21.170	0.000	
V5	1.012	0.062	16.268	0.000	• Factor loadings $(\lambda_{\rm h})$
V6	0.986	0.026	37.252	0.000	
V7	0.413	0.027	15.450	0.000	
V8	1.113	0.041	26.911	0.000	
V9	0.687	0.029	23.921	0.000	
Thresholds					
V1\$1	0.210	0.066	3.166	0.002	
V2\$1	1.753	0.066	26.398	0.000	
V3\$1	0.838	0.117	7.141	0.000	
V4\$1	-1.447	0.055	-26.417	0.000	
V5\$1	1.483	0.047	31.322	0.000	• Thresholds $(\tau_{\rm b})$
V6\$1	-0.437	0.072	-6.103	0.000	
V7\$1	-0.828	0.085	-9.764	0.000	
V8\$1	-0.343	0.085	-4.036	0.000	
V9\$1	-0.770	0.085	-9.050	0.000 🜙	
Variances					
BMATH	0.190	0.037	5.143	0.000	Factor variance (ϕ_b)
Residual Variances	3			-	
V1	0.000	0.000	999.000	999.000	
V2	0.000	0.000	999.000	999.000	
V3	0.000	0.000	999.000	999.000	
V4	0.000	0.000	999.000	999.000	
V5	0.000	0.000	999.000	999.000	Residual variances ($\theta_{\rm h}$)
V6	0.000	0.000	999.000	999.000	
₩7	0.000	0.000	999.000	999.000	Fixed at zero
VB	0.000	0.000	999.000	999.000	
V9	0.000	0.000	999.000	999.000	

Calculating the ICC of the common factors using the factor variances: $ICC = \phi_b / (\phi_b + \phi_w)$.190/(.190 + 1) = .160

Parameter estimates of Step 3a (standardized)

_				Two-Tailed		
E	stimate	S.E.	Est./S.E.	P-Value		
Within Level						
MATH BY						
V1	0.678	0.010	69.511	0.000	ר	
V2	0.603	0.013	46.510	0.000		
V3	0.725	0.011	66.376	0.000		
V4	0.489	0.018	27.837	0.000		
V5	0.711	0.022	32.924	0.000		Factor loadings (λ_{-})
V6	0.702	0.010	73.505	0.000		
V7	0.382	0.021	18.088	0.000		
V8	0.744	0.012	60.229	0.000		
V 9	0.566	0.016	35.204	0.000	J	
Variances						
MATH	1.000	0.000	999.000	999.000	1	Factor variance (ϕ_w)
Between Level						
BMATH BY						
V1	1.000	0.000	*******	0.000		
V2	1.000	0.000	999.000	999.000		
V3	1.000	0.000	*******	0.000		
V4	1.000	0.000	999.000	999.000		
V5	1.000	0.000	*******	0.000		Factor loadings ())
V6	1.000	0.000	999.000	999.000	Г	Factor loadings (Λ_b)
V 7	1.000	0.000	*******	0.000		
V8	1.000	0.000	999.000	999.000		All standardized factor
V9	1.000	0.000	*******	0.000		
					J	loadings are 1, because
Thresholds						residual variance is zero
V1\$1	0.154	0.049	3.159	0.002		
V2\$1	1.398	0.058	23.975	0.000		
V3\$1	0.577	0.076	7.597	0.000		
V4\$1	-1.262	0.053	-23.995	0.000		
V5\$1	1.042	0.059	17.758	0.000	L	$T_{1} = 1 + 1 + (-)$
V6\$1	-0.311	0.051	-6.089	0.000		I hresholds (τ_b)
V7\$1	-0.765	0.073	-10.486	0.000		
V8\$1	-0.229	0.060	-3.819	0.000		
V9\$1	-0.635	0.064	-9.987	0.000		
Variances					_	
BMATH	1.000	0.000	999.000	999.000	}	Factor variance (ϕ_b)
Residual Variances						
V1	0.000	999.000	999.000	999.000	ר	
V2	0.000	999.000	999.000	999.000		
V3	0.000	999.000	999.000	999.000		
V4	0.000	999.000	999.000	999.000		
V5	0.000	999.000	999.000	999.000	L	Desidual variances (0)
V6	0.000	999.000	999.000	999.000	Γ.	(σ_b)
V7	0.000	999.000	999.000	999.000		T ' 1 (
V8	0.000	999.000	999.000	999.000		Fixed at zero
V9	0.000	999.000	999.000	999.000	_ J _ '	

Fit statistics of Step 4 (Adding observed variables)

MODEL FIT INFORMATION								
Number of Free Parameters	26							
Chi-Square Test of Model Fit								
Value Degrees of Freedom P-Value	98.660* 87 0.1848							
* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.								
RMSEA (Root Mean Square Error Of Approximation)								
Estimate	0.003							
CFI/TLI								
CFI TLI	0.998							
χ² ₍₈₇₎ = 98.66, <i>p</i> = .18, RMSEA = .003, CF	FI = .998							

Parameter estimates of Step 4 (unstandardized)

						Two-Tailed	1	
			Estimate	S.E.	Est./S.E.	P-Value	-	
ſ	Within Le	vel						
	MATH	BY						
	V1		0.921	0.025	37.589	0.000		
	V2		0.756	0.026	29.607	0.000		
	V3		1.051	0.034	31.163	0.000		
	V4		0.562	0.027	21.063	0.000		
	V5		1.012	0.063	16.124	0.000	1	Factor loadings (λ_{-})
	V6		0.986	0.026	37.249	0.000		
	V7		0.414	0.027	15.396	0.000		
	V8		1.113	0.041	26.952	0.000		
	79		0.688	0.029	23.814	0.000	J	
	Variance	5					1	Factor verience (a)
	MATH		1.000	0.000	999.000	999.000	3	Factor variance (ϕ_w)
	Between L	evel						
	BMATH	BY					_	
	V1		0.921	0.025	37.589	0.000		
	V2		0.756	0.026	29.607	0.000		
	V3		1.051	0.034	31,163	0.000		
	V4		0.562	0.027	21.063	0.000		
	V5		1.012	0.063	16,124	0.000		Factor loadings ($\lambda_{\rm b}$)
	V6		0.986	0.026	37 249	0.000		
	77		0 414	0.027	15 396	0.000		
	VS		1 113	0.041	26 952	0.000		
	V9		0.688	0.029	23.814	0.000	J	
	BMATH	WITH						
	SHORT		0.009	0.021	0.433	0.665	1	
	STRAT	IO	-0.040	0.020	-1.965	0.049		Covariances between common
							ŀ	
	SHORT	WITH						factor and contextual variables
	STRAT	IO	0.023	0.019	1.213	0.225	J	
	Means							
	SHORT		1.518	0.059	25.785	0.000		Means of contextual variables
	STRAT	to	1.085	0.074	14.607	0.000	5	Wiealis of contextual variables
	Threshold	-1 e						
	V1\$1	10	0 210	0.066	3 167	0 002		
	V2\$1		1 753	0.067	26 354	0.002		
	V291		0.837	0.007	7 122	0.000		
	V4\$1		_1 449	0.055	-26 450	0.000		
	V5\$1		1 493	0.047	31 617	0.000		Threadholds $(-)$
	V6\$1		-0 437	0.017	-6 106	0.000	Γ	Thresholds (τ_b)
	V751		-0.437	0.072	-0.100	0.000		
	17001		0.020	0.005	4 025	0.000		
	VOSI		-0.343	0.085	-9.035	0.000		
					2.010		J	
	Variance:	3					_	
	SHORT		0.098	0.028	3.464	0.001		Variances of common factor
	STRAT	IO	0.152	0.035	4.364	0.000		
	BMATH		0.190	0.037	5.143	0.000		and contextual variables
	Residual	Variances					1	
	V1		0.000	0.000	999.000	999.000	1	
	V2		0.000	0.000	999.000	999.000		
	V3		0.000	0.000	999.000	999.000		
	V4		0.000	0.000	999.000	999.000		\mathbf{D} and \mathbf{I} and \mathbf{I} are the set of (0)
	V5		0.000	0.000	999.000	999.000		Residual variances (θ_b)
	V6		0.000	0.000	999.000	999.000		-
	V7		0.000	0.000	999.000	999.000		
	V8		0.000	0.000	999.000	999.000		
	V9		0.000	0.000	999.000	999.000	J	

Two-Tailed S.E. Est./S.E. Estimate **P-Value** Within Level MATH BY 0.678 0.010 69.506 0.000 V1 0.013 46.525 0.000 V2 0.603 V3 0.725 0.011 65.592 0.000 V4 0.490 0.018 27.724 0.000 **V**5 0.711 0.022 32.629 0.000 Factor loadings (λ_w) V6 0.702 0.010 73.454 0.000 V7 0.382 0.021 18.031 0.000 VR 0.744 60.310 0.012 0.000 V9 0.567 0.016 35.070 0.000 Variances Factor variance (ϕ_w) 999.000 MATH 1.000 0.000 999.000 Between Level BMATH BY 999.000 999.000 1,000 0.000 V1 V2 1.000 0.000 999.000 999.000 V3 1.000 0.000 999.000 999.000 V4 1.000 0.000 999.000 999.000 Factor loadings ($\lambda_{\rm b}$) ******* **V**5 1.000 0.000 0.000 0.000 ******** V61.000 0.000 999.000 V7 1.000 0.000 999.000 V8 0.000 999.000 1.000 999.000 V9 1.000 0.000 ******* 0.000 BMATH WITH SHORT 0.066 0.150 0.442 0.659 STRATTO -0.2360.112 -2.098 0.036 Correlations between common factor and contextual variables SHORT WITH STRATIO 0.189 0.147 1.292 0.196 Means SHORT 4.849 0.635 7.636 0.000 Means of contextual variables STRATIO 2.786 0.281 9,910 0.000 Thresholds 0.154 0.049 0.002 V1\$1 3.159 0.000 V2\$1 1.398 0.058 23.975 V3\$1 0.577 0.076 0.000 7.597 V4\$1 -1.262 0.053 -23.995 0.000 V5\$1 17.758 0.000 Thresholds $(\tau_{\rm h})$ 1.042 0.059 V6\$1 -0.311 0.051 -6.089 0.000 V7\$1 -0.765 0.073 -10.4860.000 V8\$1 -0.2290.060 -3.8190.000 V9\$1 -0.635 0.064 -9.9870.000 Variances 1.000 0.000 999.000 999.000 SHORT Variances of common factor STRATIO 1.000 0.000 999.000 999.000 and contextual variables BMATH 1.000 0.000 999.000 999.000 Residual Variances 999.000 0.000 999.000 999.000 V1 V2 0.000 999.000 999.000 999.000 V3 0.000 999.000 999.000 999.000 V4 0.000 999.000 999.000 999.000 Residual variances $(\theta_{\rm b})$ 999.000 **V**5 0.000 999.000 999.000 V6 0.000 999,000 999.000 999.000 **V7** 0.000 999.000 999.000 999.000 V8 0.000 999.000 999.000 999.000 V9 0.000 999.000 999.000 999.000

Parameter estimates of Step 4 (standardized)